

## FIELD OF THE INVENTION

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System (POTS) splitter generally must be installed at the user end to separate the voice frequencies and the ADSL frequencies. The G.lite version of ADSL, also known as the ADSL lite, Universal ADSL or splitterless ADSL, gets around the splitter requirement by having all phones plug into low-pass filters that remove the ADSL high frequencies from the voice transmissions. ADSL is available in two modulation schemes: the Discrete Multi-tone (DMT) or Carrierless Amplitude Phase (CAP).

In DMT-based DSL modems, the selected bandwidth of 1.104 MHz is divided into bins and the data bits are used for Quadrature Amplitude Modulation in each bin. During the initialization period, a channel SNR estimation phase is employed to transmit a known pseudo-random noise (PRN) sequence while the receiver computes the channel characteristics from the received signal. The characteristics are computed in the form of a  $g_k \cdot N_k^{-1}$  ratio, where  $g_k$  is the channel gain (attenuation,  $|H(k)|^2$ ) in frequency band  $k$ , and  $N_k$  is the noise power in band  $k$ . Prior art has disclosed a number of algorithms for determining the power distribution across the full frequency bandwidth for maximum data rate. The optimum approach for Additive White Gaussian Noise (AWGN), has been proved to be a 'water pouring' algorithm of power distribution, where the  $g_k \cdot N_k^{-1}$  profile is considered to be equivalent to the 'terrain' and the available power budget is likened to 'water that is poured' on the terrain. In this analogy, the water depth at position  $k$  is equivalent to the power allocated to the frequency bin  $k$ .

The following analysis provides a brief description of this approach. As is known to those skilled in the art, the relationship between the number of bits in a frequency bin and the

power needed to transmit that number of bits, for a specified bit error rate (BER) at the receiver for which  $g_k \cdot N_k^{-1}$  is the measured channel characteristic, is given by the following expression:

$$b_k = \log_2 \left[ 1 + \frac{3 \cdot g'_k \cdot E_k}{K \cdot (N_k)} \right] \quad k=1 \dots 256 \quad \text{Eq. 1}$$

where

$b_k$  = No. of bits in frequency bin  $k$

$E_k$  = Power required in bin  $k$  to transmit the  $b_k$  bits

5  $\frac{g'_k}{N_k}$  = Measured channel attenuation to noise power ratio in bin  $k$

$N_k$  = Noise power in bin  $k$

$$K = \left[ Q^{-1} \left( \frac{P_e}{N_e} \right) \right]^2 \text{ where } 2 \leq \left[ N_e = 4 \cdot \left( 1 - \frac{1}{\sqrt{2^{b_k}}} \right) \right] \leq 4 \text{ for } 2 \leq b_k$$

Given the expression in Eq. 1, the power needed to transmit  $b_k$  bits in bin  $k$  can be obtained by inverting the expression by ignoring the dependence of  $N_e$  on  $b_k$ . (Prior art has shown that approximating  $N_e$  by a constant between 2 and 4 has a negligible effect on the overall data capacity.)

$$E_k = \frac{KN}{3g'_k} (2^{b_k} - 1) = \frac{N}{g_k} (2^{b_k} - 1) \text{ where } g_k = \frac{3g'}{K}$$

The problem of power allocation consists of distributing the available power budget over the 256-bins so that the capacity as defined by  $\sum_{k=1}^{256} b_k$  is maximized. The allocation must be performed within constraints of the DSL modems that are subject to a power mask constraint that limits the maximum power that may be allocated to each bin.

The solution to the 2-tone power allocation problem is known in the art. The available power is distributed optimally over two bins to maximize the 2-bin capacity of  $b_1 + b_2$ . In order to perform the 256-bin power allocation, the prior art proposes an iterative approach to solve the

“water-pouring” problem. However, such a solution for the 256-bin power allocation results in noisy bins. Thus, there is a need for a method, system and computer medium for assigning data bits to bins for simplex transmission while minimizing noise.

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## SUMMARY OF THE INVENTION

The present invention provides a method, apparatus, computer readable medium and modem for allocation of an available power to Discrete Multi-Tone (DMT) frequency tones in a DMT-based Digital Subscriber Line (DSL) modem. The steps typically include: (1) initializing the DMT-based DSL modem by calculating aggregate values of channel attenuation, noise  
10 power, and power mask; (2) pre-filtering to flag noisy bins that are unable to support a minimum number of bits with a maximum power available for transmission in a bin; and (3) using a repeated-bisection splitting scheme to allocate the available power substantially optimally among a plurality of bands for DMT frequency tones, where bins flagged in the pre-filtering step are excluded.

15 In one embodiment, the repeated-bisection splitting scheme may include the steps of: splitting the available power optimally between the lower and the upper halves of a 1.104-MHz bandwidth to form two power fractions for two substantially 552 KHz wide bands; splitting each of the two power fractions substantially optimally between two halves of each of the substantially 552 KHz wide bands to form four power fractions for four substantially 276 KHz  
20 wide bands; splitting each of the four power fractions optimally between two halves of each of the four substantially 276 KHz wide bands to form eight power fractions for eight substantially 138 KHz wide bands; splitting each of the eight power fractions optimally between two halves

of each of the eight substantially 138 KHz wide bands to form sixteen power fractions for sixteen substantially 69 KHz wide bands; splitting each of the sixteen power fractions optimally between two halves of each of the sixteen substantially 69 KHz wide bands to form thirty-two power fractions for thirty-two substantially 34.5 KHz wide bands; splitting each of the thirty-  
5 two power fractions optimally between two halves of each of the thirty-two substantially 34.5 KHz wide bands to form sixty-four power fractions for sixty-four substantially 17.25 KHz wide bands; splitting each of the sixty-four power fractions optimally between two halves of each of the sixty-four 17.25 KHz wide bands to form one hundred twenty eight power fractions for one hundred twenty eight substantially 8.625 KHz wide bands; and splitting each of the one hundred  
10 twenty eight power fractions optimally between two halves of each of the one hundred twenty eight, 8.625 KHz wide bands to form two hundred fifty six power fractions to form two hundred fifty six substantially 4.3125 KHz wide bands.

The initializing step may include calculating aggregate parameters needed for the splitting steps while excluding bins flagged in the pre-filtering step.

15 Each of  $2^j$  elements of a noise power vector to be used at step- $j$  may be calculated as a sum of noise power values in bins aggregated for the step- $j$ , where  $j=1, \dots, 8$ . Each of  $2^j$  elements of a channel attenuation vector that is to be used at step- $j$  may be calculated as an average of channel attenuation values in bins aggregated for step- $j$ , where  $j=1, \dots, 8$ . Each of  $2^j$  elements of a power mask vector that is to be used at step- $j$  may be calculated as a sum of  
20 power mask values in bins aggregated for step- $j$ , where  $j=1, \dots, 8$ .

The available power may be allocated to  $2^n$  tones where  $n$  is a preselected integer. Here, initializing includes calculating aggregate parameter values of channel attenuation, noise power, and power mask for  $n$  subsequent steps.

The apparatus of the present invention allocates the available power budget to a plurality of Discrete Multi-Tone (DMT) frequency tones using a repeated-bisection of power scheme to partition the available power over the plurality of DMT frequency tones in a DMT-based Digital Subscriber Line (DSL) modem. The apparatus includes an initialization unit, a pre-filtering unit, and a repeated-bisection power splitting and allocation unit. The initialization unit initializes the DMT-based DSL modem by calculating aggregate parameter values of channel attenuation, noise power, and power mask. The pre-filtering unit is coupled to the initialization unit and is used for pre-filtering to flag noisy bins that are unable to support a minimum number of bits with a maximum power available for transmission in a bin. The pre-filtering step is utilized to ensure that narrow-band energy from radio sources, power lines, and other line-spectrum-producing interferers does not distort the power-allocation process by “wasting” power in bins which overlay these interferers. The repeated-bisection power splitting and allocation unit is coupled to the pre-filtering unit and is used for implementing the repeated bisection of power scheme of the invention to allocate available power substantially optimally among a plurality of bands for DMT frequency tones, where bins flagged by the pre-filtering unit are excluded. Where desired, the initialization unit may receive notification of the flagged noisy bins from the pre-filtering unit and exclude the flagged noisy bins.

## BRIEF DESCRIPTION OF THE DRAWINGS

The foregoing summary of the invention, as well as the following detailed description of preferred embodiments, is better understood when read in conjunction with the accompanying drawings, which are included by way of example, and not by way of limitation with regard to  
 5 the claimed invention.

Figure 1 is a schematic representation of steps, splits and frequency bands in a multi-tone power splitting scheme in accordance with the present invention.

Figure 2 is a flow diagram of one embodiment of steps in accordance with the method of the present invention.

10 Figure 3 is a block diagram of a modem having an apparatus for allocation of an available power to Discrete Multi-Tone frequency tones in a Discrete Multi-Tone frequency-based Digital Subscriber Line in accordance with the present invention.

Figure 4 is a flow chart showing two embodiments of steps in accordance with the method of the present invention.

## DETAILED DESCRIPTION OF THE PREFERRED EMBODIMENTS

The present invention utilizes a "repeated-bisection" scheme to extend the 2-bin solution to 256-bins instead of using the iterative scheme suggested in the prior art. The prior art considers the simple case in which the power is to be distributed over two bins. The two-bin  
 20 capacity,  $b_1 + b_2$  is maximized with respect to the power to be allocated to bin-1 as  $(E_r)$ , when the power is  $E$  and the power to be allocated to bin-2 is  $E - E_r$ . It can be shown that the optimum power allocation to bin 1 is given by the following expression.

$$E_r = \frac{1}{2}(E + N_2 \cdot g_2^{-1} - N_1 \cdot g_1^{-1}) \quad \text{Eq. 2}$$

This solution is derived for the two-tone simplex transmission. It is also the water-filling solution, e.g., if the noise power and attenuation coefficients in the two bins are equal, the two terms cancel each other and the power is divided equally between the two bins. Similarly, the smaller the ratio of noise power to attenuation coefficient is in bin 1, the larger the value of  $E_r$  is, which in turn, allocates greater power to bin 1. The above expression may be simplified by denoting  $M_i = N_i \cdot g_i^{-1}$  for  $i = 1, 2$  and  $t_{21} = M_2^{-1} - M_1^{-1}$ :

$$E_r = 0.5 \cdot (E + t_{21}) \text{ and } E - E_r = 0.5 \cdot (E - t_{21}) = 0.5 \cdot (E + t_{12}) \quad \text{Eq. 3}$$

Discrete Multi-tone-based Digital Subscriber Loop (DSL) modems require a scheme for allocating bits and power to the discrete tones during the initialization period. As shown in Figure 2, the initialization parameters are computed, and pre-filtering is used to remove bins that cannot support the minimum number of bits. Then the repeated-bisection splitting scheme of the present invention partitions available power over all the frequency bins to which power needs to be assigned for DMT-transmission. The power allocation is performed with the objective of maximizing the total data rate.

### Extension to Multi-Bins

In the present invention, a repeated bisection scheme is used for extending the two-tone solution to the 256-tone case. The scheme operates by first bisecting the 1.104 MHz band into halves to solve the first '2-tone' problem. In the second split, two problems, each a different 2-tone problem, are solved. In split 3, four problems, each a different 2-tone problem, are solved. This process of progressively splitting each band in halves is continued for 8 splits, until power



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is allocated to each of the 256 bins. The repeated bisection scheme 206, 208, 210, 210, 212, 214, 216, 218 is shown schematically in Figures 1 and 2. The number of bands that are produced as a result of split- $i$  is equal to  $2^i$  and the bandwidth of each band at split- $i$  is equal to  $\frac{1104}{2^i}$  KHz. Thus, at split 1, each band is 552 KHz wide and at the 8<sup>th</sup> split, each band

5 corresponds to the DMT frequency bin. Figure 1 also shows the number of 2-tone solutions that must be computed at each split. At split-1, one 2-tone solution generates the partition of the budgeted power over each of two 552 KHz frequency bands. Each of these two power fractions is split into two parts in split-2 to yield power allocations to four 276 KHz frequency bands. Figure 1 shows that the maximum number of 2-tone solutions that may have to be computed is  
10 equal to 255. The following explains the procedure for calculating the values of the channel attenuation and noise PSD profiles at each split.

#### **Pre-filtering to eliminate noisy bins**

In a pre-filtering step, the invention provides for calculating the number of bits that can be received in a bin with the desired bit error rate (BER), when the tone is transmitted with a  
15 power level that is equal to that specified by the power mask value at the frequency bin. If the calculated value is less than the minimum number of bits that can be allocated to that bin, then the bin is flagged for exclusion from the subsequent power allocation steps.

#### **Input power at each split**

In the first split, the power budgets allocated are allocated to the two halves. For  
20 example, the 0.1-Watt budgeted power would be split between the upper and the lower halves of the entire 1.104 MHz band.

In the second split, the input power to each of the two portions would be the partial power allocated to the corresponding half-band in split 1. For example, if 75 mWatt is assigned to the lower half-band and the rest of the 25 mWatts is allocated to the upper half-band, then the power inputs to the two 2-tone portions is 75 and 25 mWatts, respectively.

- 5 In the third split, the power input to each of the four portions is the partial power allocated to the corresponding quarter-band in split 2.

As shown in Figure 1, step-8 consists of performing 128 solutions of the 2-bin allocation portions. Thus, split 8 is the only part of the scheme that uses the normal DMT bins. All of the earlier 7 steps require system parameters for progressively wider frequency bands. In fact, the  
 10 'tone' to which power is allocated in step- $i$  ( $i = 1 \dots 8$ ) is of a bandwidth that corresponds to that of  $2^{8-i}$  frequency bins. Hence, the appropriate channel parameters must be calculated for each step.

- 15 
$$\begin{aligned} &\text{No. of 2-tone solutions at step-}i = 2^{i-1} \\ &\text{No. of frequency bands produced at step } i = 2^i \\ &j \triangleq \text{Band Index at step-}i = 1 \dots 2^i \end{aligned}$$

#### Noise power at each split

- Noise power in band index  $j$  should be the total noise power in that band. If  
 20  $N_k$  represents the noise power in bin  $k$  ( $k=1 \dots 256$ ), then the noise power in band  $j$  at step  $i$  may be calculated as follows and stored for use in the 7-steps of the band-splitting scheme. The calculation of the integrated noise power values needs to be performed just once, and it adds 254-array elements to the storage requirement. Note that the summations for average noise calculation are performed over only those bins that can support at least the minimum number of

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bits with the maximum power that can be allocated to one bin as determined in the pre-filtering step above.

$$N(i, j) = \sum_{k \in \text{band}_j} N_k \text{ where, } k = 1 \dots 2^i \quad \text{Eq. 4}$$

#### Channel attenuation parameters at each split

- 5        The channel attenuation coefficient in bin- $k$  in each band,  $g_k$ , is the average value for the band. The summations for the calculation of average channel attenuation are performed over only those bins that were not excluded by the pre-filtering step.

$$g(i, j) = \frac{1}{2^{8-i}} \cdot \left( \sum_{k \in \text{band}_j} g_k \right) \text{ where } j = 1 \dots 2^i \quad \text{Eq. 5}$$

#### 10        Power mask constraint at each split

- The power mask constraint may be applied by calculating the implied mask constraint for each of the wider bands used in splits 1 through 7. Power mask in band index  $j$  should be the total power mask in that band. If  $Pmask_k$  represent the power mask values in bin  $k$  ( $k=1 \dots 256$ ), then the power mask in band  $j$  at step  $i$  can be calculated and stored for use in the
- 15    7-steps of the band-splitting scheme. The calculation of the integrated power mask values needs to be performed just once, and it adds 254-array elements to the storage requirement. The ADSL parameters are such that the total integrated power mask exceeds the power budget. Hence, a signal is not intended to be as large as to reach the power mask at all the frequencies. Therefore, the integrated values of the power mask for the wider bins are compared with the
- 20    total power budget and the integrated power mask is set as the minimum of the integrated value and the total power budget.

$$P_{mask}(i, j) = \min \left( \sum_{k \in \text{band}_j} P_{mask_k}, P_{budget} \right) \text{ where } j = 1 \dots 2^i$$

Figure 3 is a block diagram of a modem 302 having an apparatus 304 for allocating an available power to a plurality of Discrete Multi-Tone (DMT) frequency tones using a repeated-bisection of power scheme to partition the available power over the plurality of DMT frequency tones in a DMT-based Digital Subscriber Line (DSL) in accordance with the present invention. The apparatus 304 includes an initialization unit 306, a pre-filtering unit 308 and a repeated-bisection power splitting and allocation unit 310. The initialization unit 304 is used for initializing the DMT-based DSL modem by calculating aggregate parameter values of channel attenuation, noise power, and power mask. The pre-filtering unit 308 is coupled to the initialization unit 306 and is used for pre-filtering to flag noisy bins that are unable to support a minimum number of bits with a maximum power available for transmission in a bin. The repeated-bisection power splitting and allocation unit 310 is coupled to the pre-filtering unit 308 and is utilized for using the repeated-bisection of power scheme to allocate the available power substantially optimally among a plurality of bands for DMT frequency tones. As shown by the solid lines, the available power may be sent to the initialization unit 306, which sends its output to the pre-filtering unit 308, which sends its output to the repeated-bisection power splitting and allocation unit 310. The actions taken in the units are as described below in the corresponding method steps.

In one embodiment, the repeated-bisection splitting unit 310 may implement the repeated-bisection of power scheme by: splitting the available power optimally between the lower and the upper halves of a 1.104-MHz bandwidth to form two power fractions for two

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substantially 552 KHz wide bands; splitting each of the two power fractions substantially  
optimally between two halves of each of the substantially 552 KHz wide bands to form four  
power fractions for four substantially 276 KHz wide bands; splitting each of the four power  
fractions optimally between two halves of each of the four substantially 276 KHz wide bands to  
5 form eight power fractions for eight substantially 138 KHz wide bands; splitting each of the  
eight power fractions optimally between two halves of each of the eight substantially 138 KHz  
wide bands to form sixteen power fractions for sixteen substantially 69 KHz wide bands;  
splitting each of the sixteen power fractions optimally between two halves of each of the sixteen  
substantially 69 KHz wide bands to form thirty-two power fractions for thirty-two substantially  
10 34.5 KHz wide bands; splitting each of the thirty-two power fractions optimally between two  
halves of each of the thirty-two substantially 34.5 KHz wide bands to form sixty-four power  
fractions for sixty-four substantially 17.25 KHz wide bands; splitting each of the sixty-four  
power fractions optimally between two halves of each of the sixty-four 17.25 KHz wide bands  
to form one hundred twenty eight power fractions for one hundred twenty eight substantially  
15 8.625 KHz wide bands; and splitting each of the one hundred twenty eight power fractions  
optimally between two halves of each of the one hundred twenty eight, 8.625 KHz wide bands  
to form two hundred fifty six power fractions to form two hundred fifty six substantially 4.3125  
KHz wide bands.

Alternatively, the available power may be sent to the pre-filtering unit 308 first, whose  
20 output is sent to the initialization unit 306, and the output of the initialization unit may be sent  
to the repeated-bisection power splitting and allocation unit 310. In this embodiment, the  
initialization unit 306 is coupled to the pre-filtering unit 308 which receives the available

power, such that the initialization unit 306 receives notification of the bins that are flagged, and then calculates aggregate parameter values needed to implement the repeated-bisection of power scheme and excludes bins flagged by the pre-filtering unit.

In one embodiment, each of  $2^j$  elements of a noise power vector to be used at step- $j$  of the repeated-bisection of power scheme may be calculated as a sum of noise power values in bins aggregated for the step- $j$ , where  $j=1, \dots, 8$ . Similarly, each of  $2^j$  elements of a channel attenuation vector that is to be used at step- $j$  of the repeated-bisection of power scheme may be calculated as an average of channel attenuation values in bins aggregated for step- $j$ , where  $j=1, \dots, 8$ . Also, each of  $2^j$  elements of a power mask vector that is to be used at step- $j$  of the repeated-bisection of power scheme may be calculated as a sum of power mask values in bins aggregated for step- $j$ , where  $j=1, \dots, 8$ .

The available power may be selected to be allocated to  $2^n$  tones where  $n$  is a preselected integer. Typically, in this embodiment, the initialization unit 306 initializes the modem by calculating aggregate parameter values of channel attenuation, noise power, and power mask for  $n$  subsequent steps.

Figure 4 shows a method for allocation of an available power to Discrete Multi-Tone (DMT) frequency tones in a DMT-based Digital Subscriber Line (DSL) modem in accordance with the present invention. The method typically includes the steps of: initializing 402 the DMT-based DSL modem by calculating aggregate values of channel attenuation, noise power, and power mask; pre-filtering 404 to flag noisy bins that are unable to support a minimum number of bits with a maximum power available for transmission in a bin; and using a repeated-bisection splitting scheme 406 to allocate the available power substantially optimally among a

plurality of bands for DMT frequency tones, wherein, where bins flagged in the pre-filtering process and not yet excluded are excluded by the scheme.

The method may implement the repeated-bisection splitting scheme by: splitting the available power optimally between the lower and the upper halves of a 1.104-MHz bandwidth

5 to form two power fractions for two substantially 552 KHz wide bands; splitting each of the two power fractions substantially optimally between two halves of each of the substantially 552 KHz wide bands to form four power fractions for four substantially 276 KHz wide bands; splitting each of the four power fractions optimally between two halves of each of the four substantially 276 KHz wide bands to form eight power fractions for eight substantially 138 KHz

10 wide bands; splitting each of the eight power fractions optimally between two halves of each of the eight substantially 138 KHz wide bands to form sixteen power fractions for sixteen substantially 69 KHz wide bands; splitting each of the sixteen power fractions optimally between two halves of each of the sixteen substantially 69 KHz wide bands to form thirty-two power fractions for thirty-two substantially 34.5 KHz wide bands; splitting each of the thirty-

15 two power fractions optimally between two halves of each of the thirty-two substantially 34.5 KHz wide bands to form sixty-four power fractions for sixty-four substantially 17.25 KHz wide bands; splitting each of the sixty-four power fractions optimally between two halves of each of the sixty-four 17.25 KHz wide bands to form one hundred twenty eight power fractions for one hundred twenty eight substantially 8.625 KHz wide bands; and splitting each of the one hundred

20 twenty eight power fractions optimally between two halves of each of the one hundred twenty eight, 8.625 KHz wide bands to form two hundred fifty six power fractions to form two hundred fifty six substantially 4.3125 KHz wide bands.

Where the pre-filtering step 404 precedes the initializing step 402 (dashed lines), the initializing step 402 may provide for calculating aggregate parameters needed while excluding noisy bins that are flagged as unable to support a minimum number of bits with a maximum power available for transmission in a bin.

- 5 Each of  $2^j$  elements of a noise power vector to be used at step- $j$  may be calculated as a sum of noise power values in bins aggregated for the step- $j$ , where  $j=1,\dots,8$ . Also, each of  $2^j$  elements of a channel attenuation vector that is to be used at step- $j$  may be calculated as an average of channel attenuation values in bins aggregated for step- $j$ , where  $j=1,\dots,8$ . In addition, each of  $2^j$  elements of a power mask vector that is to be used at step- $j$  may be calculated as a sum of
- 10 power mask values in bins aggregated for step- $j$ , where  $j=1,\dots,8$ .

Generally, the available power is allocated to  $2^n$  tones where  $n$  is a preselected integer. Initializing typically includes calculating aggregate parameter values of channel attenuation, noise power, and power mask for  $n$  subsequent steps.

- Clearly, the method of the invention may be implemented by digital signal processor, a
- 15 microprocessor, a general processor, or other circuitry arranged to implement the steps of the method.

- The present invention may be implemented by a computer readable medium such as, for example, a memory, computer disk, or the like, having computer-executable instructions for allocation of an available power to Discrete Multi-Tone (DMT) frequency tones in a DMT-
- 20 based Digital Subscriber Line (DSL) modem, wherein the computer-executable instructions include the steps of: initializing the modem by calculating aggregate values of channel attenuation, noise power, and power mask; pre-filtering to flag noisy bins that are unable to



support a minimum number of bits with a maximum power available for transmission in a bin; and using a repeated-bisection splitting scheme to allocate the available power substantially optimally among a plurality of bands for DMT frequency tones. The exclusion of the flagged bins may be accomplished in the initializing step or, alternatively, in the step of using the  
5 repeated-bisection splitting scheme. Where desired, the repeated-bisection splitting scheme may include the steps enumerated above.

Each of  $2^j$  elements of a noise power vector to be used at step- $j$  of the repeated-bisection splitting scheme may be calculated as a sum of noise power values in bins aggregated for the step- $j$ , where  $j=1,\dots,8$ . Similarly, each of  $2^j$  elements of a channel attenuation vector  
10 that is to be used at step- $j$  of the repeated-bisection splitting scheme may be calculated as an average of channel attenuation values in bins aggregated for step- $j$ , where  $j=1,\dots,8$ . Also, each of  $2^j$  elements of a power mask vector that is to be used at step- $j$  of the repeated-bisection splitting scheme may be calculated as a sum of power mask values in bins aggregated for step- $j$ , where  $j=1,\dots,8$ . The available power is generally allocated to  $2^n$  tones where  $n$  is a preselected  
15 integer. Initializing typically includes calculating aggregate parameter values of channel attenuation, noise power, and power mask for  $n$  subsequent steps.

Although the present invention has been described in relation to particular preferred embodiments thereof, many variations, equivalents, modifications and other uses will become apparent to those skilled in the art. It is preferred, therefore, that the present invention be  
20 limited not by the specific disclosure herein, but only by the appended claims.

What is claimed is: